

---

# **The Continuing Miracle of Information Storage Technology**

**Paul H. Siegel  
Director, CMRR  
University of California, San Diego**

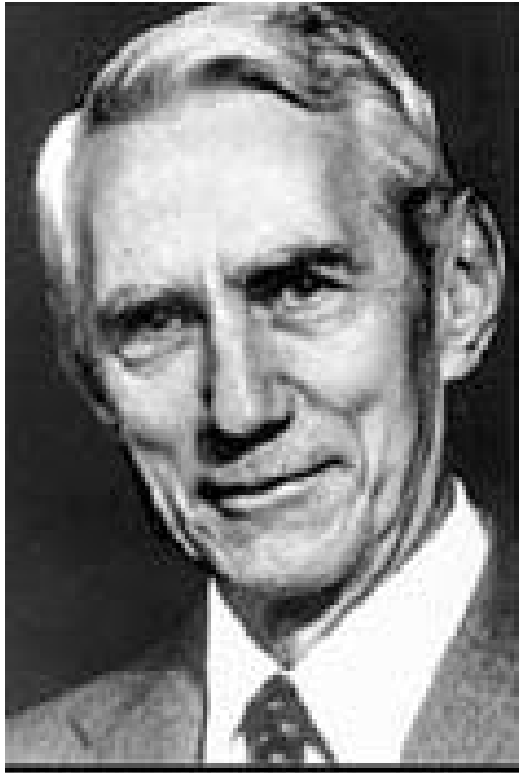
# *Outline*

---

- **The Shannon Statue**
- **A Miraculous Technology**
- **Information Theory and Information Storage**
  - ◆ **A Tale of Two Capacities**
- **Conclusion**

# *Claude E. Shannon*

---



**Claude Elwood Shannon**  
1916 - 2001



# *Acknowledgments*

---

- **For the statue - from conception to realization:**
  - **IEEE Information Theory Society**
  - **Prof. Dave Neuhoff (University of Michigan)**
  - **Eugene Daub, Sculptor**
  
- **For bringing it to CMRR:**
  - **Prof. Jack K. Wolf**

## *How Jack Did It*

---

- **6 casts of the statue**
- **Spoken for:**
  - 1. Shannon Park, Gaylord, Michigan**
  - 2. The University of Michigan**
  - 3. Lucent Technologies – Bell Labs**
  - 4. AT&T Research Labs**
  - 5. MIT**
- **Jack's idea: “6. CMRR”**

# *The Inscription*

---

**CLAUDE ELWOOD SHANNON**

1916 – 2001

**FATHER OF INFORMATION THEORY**

**HIS FORMULATION OF THE MATHEMATICAL  
THEORY OF COMMUNICATION PROVIDED  
THE FOUNDATION FOR THE DEVELOPMENT OF  
DATA STORAGE AND TRANSMISSION SYSTEMS  
THAT LAUNCHED THE INFORMATION AGE.**

**DEDICATED OCTOBER 16, 2001**

**EUGENE DAUB, SCULPTOR**

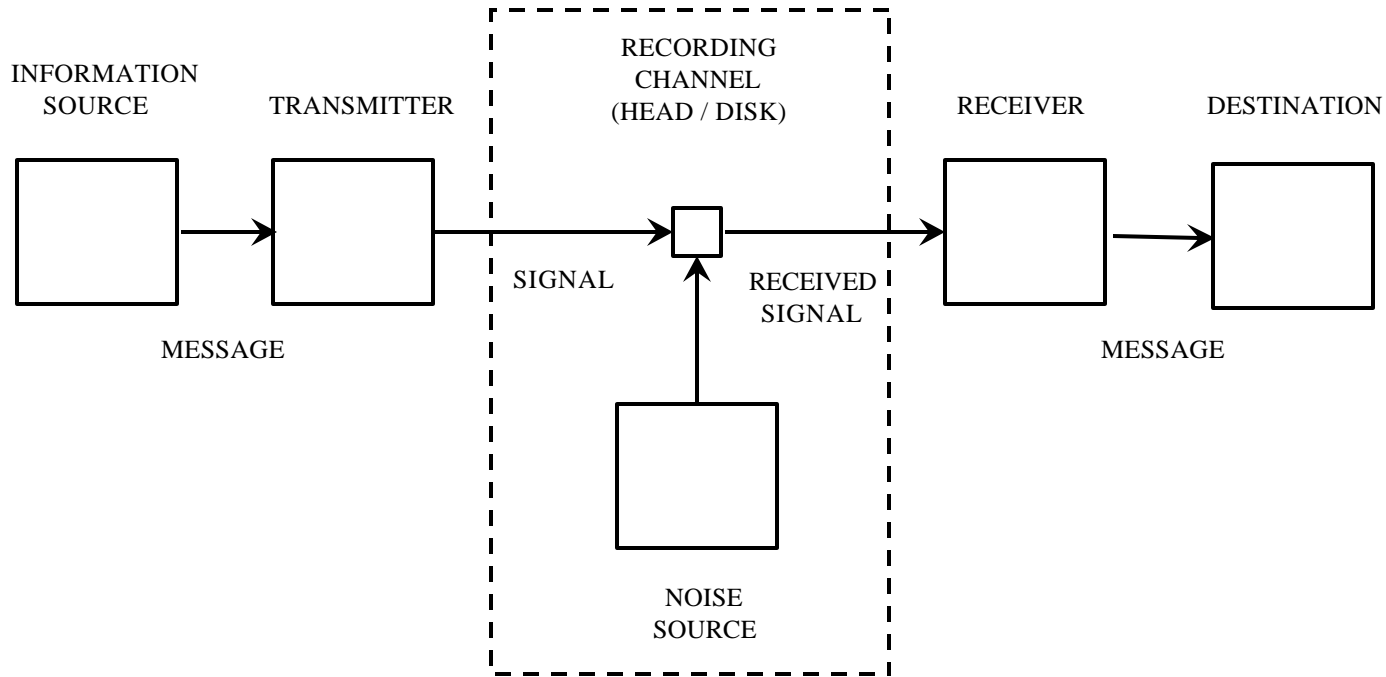
# *Data Storage and Transmission*

---

- **A data transmission system communicates information through space, i.e.,  
“from here to there.”**
- **A data storage system communicates information through time, i.e.,  
“from now to then.”**

**[Berlekamp, 1980]**

# Figure 1 (for Magnetic Recording)



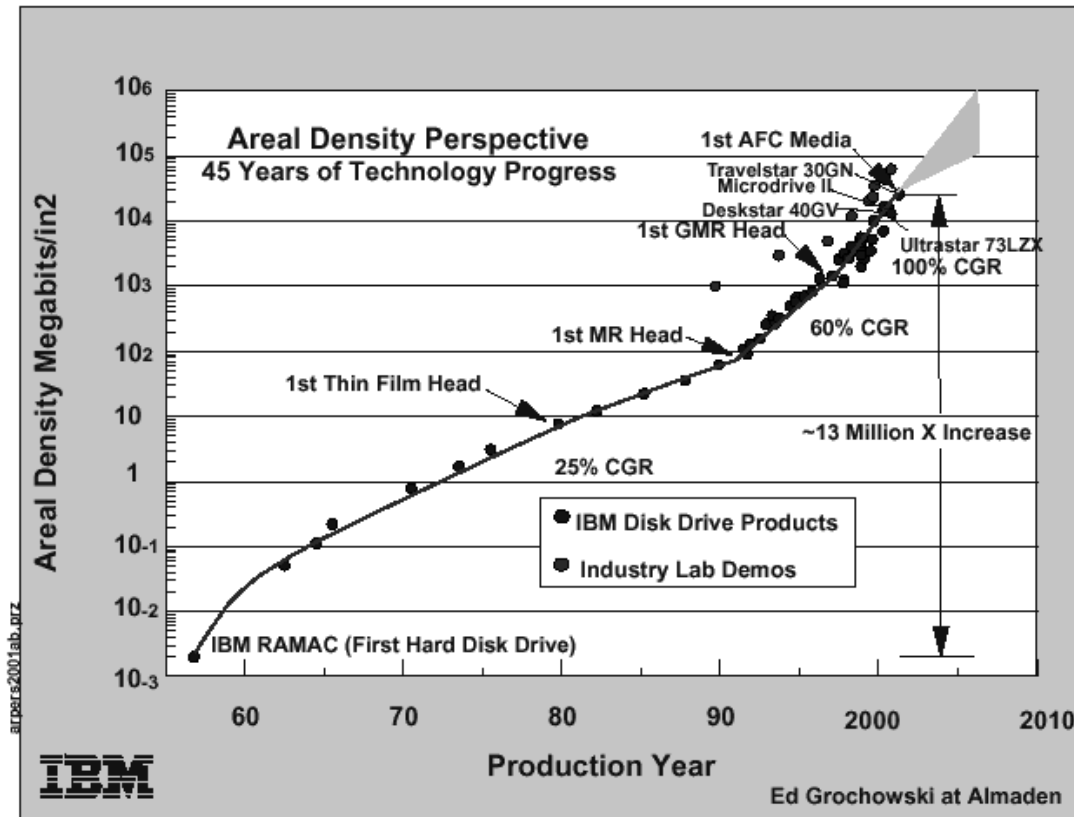
- **Binary-input**
- **Inter-Symbol Interference (ISI)**
- **Additive Gaussian Noise**

## *A Miraculous Technology*

---

- **Areal Density Perspective – 45 Years of Progress**
- **Average Price of Storage**

# Areal Density Perspective





# *The Formula on the “Paper”*

---

**Capacity of a discrete channel with noise [Shannon, 1948]**

$$C = \text{Max} (H(x) - H_y(x))$$

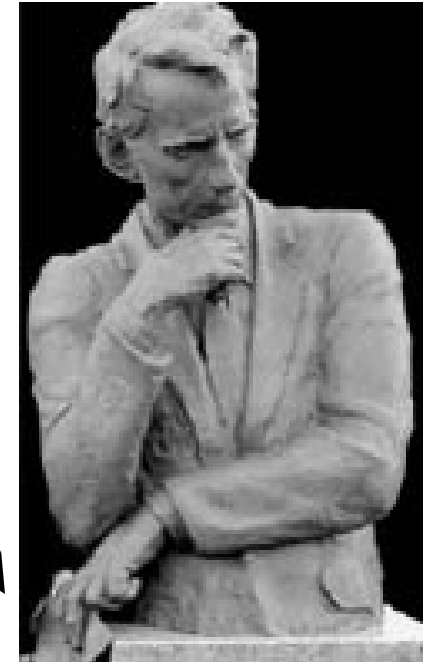
**For noiseless channel,  $H_y(x)=0$ , so:**

$$C = \text{Max} H(x)$$

**Gaylord, MI:  $C = W \log (P+N)/N$**

**Bell Labs: no formula on paper**

**(“ $H = -p \log p - q \log q$ ” on plaque)**

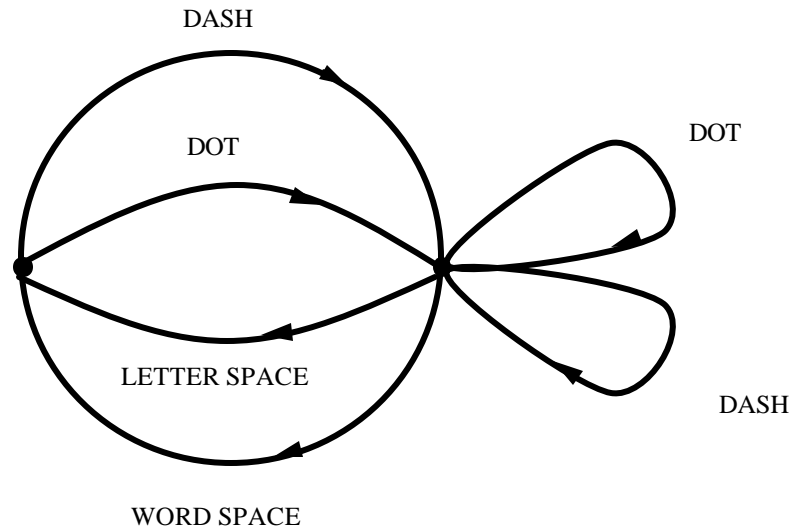


# *Discrete Noiseless Channels (Constrained Systems)*

---

- A constrained system  $S$  is the set of sequences generated by walks on a labeled, directed graph  $G$ .

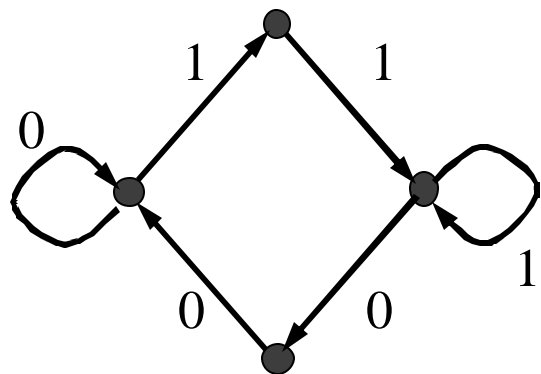
**Telegraph channel constraints [Shannon, 1948]**



# Magnetic Recording Constraints

## Runlength constraints

(“finite-type”: determined by finite list  $F$  of forbidden words)

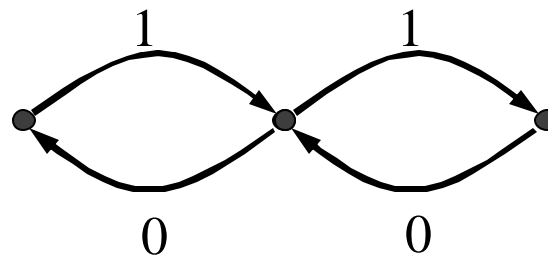


Forbidden words  $F = \{101, 010\}$

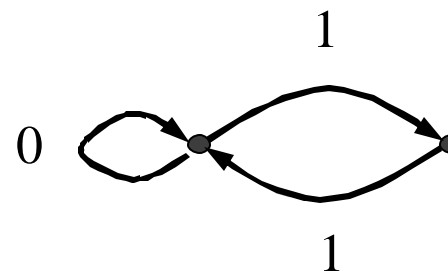
## Spectral null constraints

(“almost-finite-type”)

Biphase



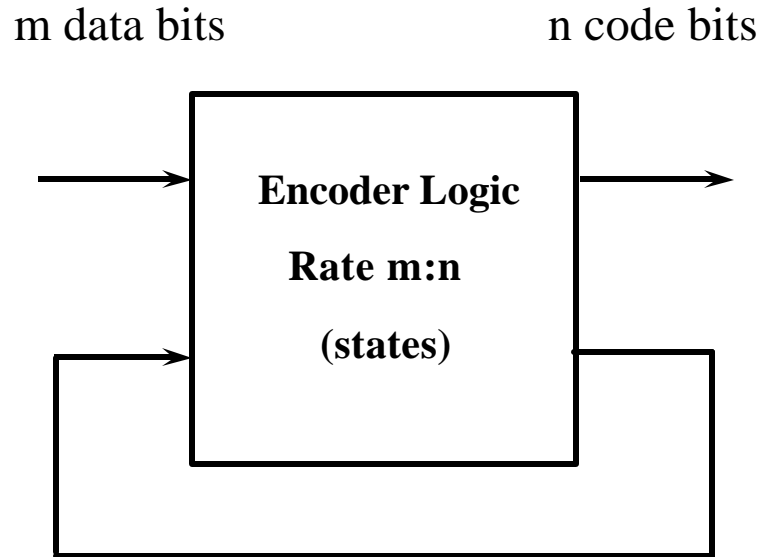
Even



# Practical Constrained Codes

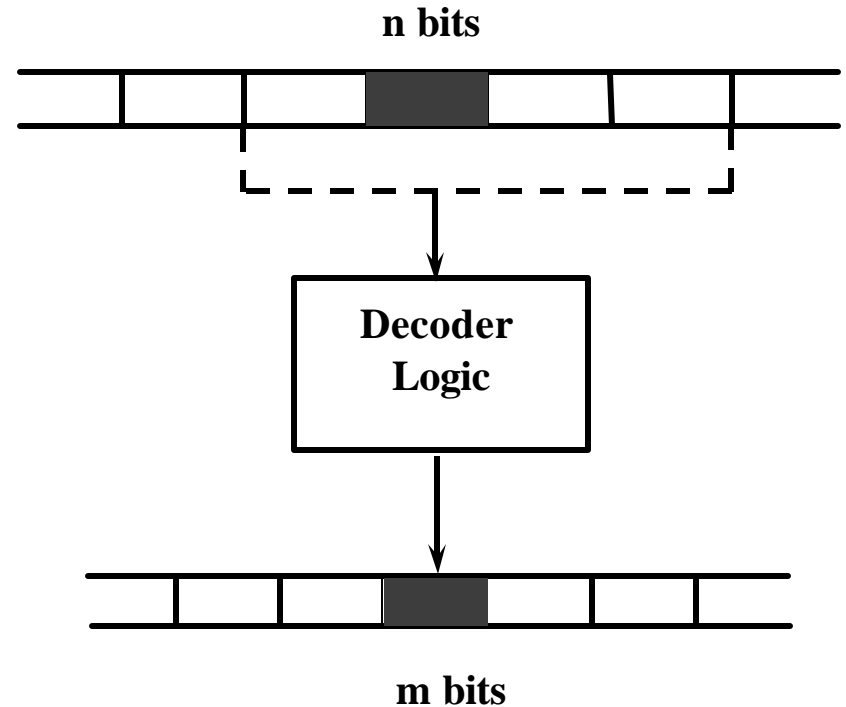
## Finite-state encoder

(from binary data into S)



## Sliding-block decoder

(inverse mapping from S to data)



We want: high rate  $R=m/n$   
low complexity

# Codes and Capacity

---

- How high can the code rate be?
- Shannon defined the capacity of the constrained system  $S$ :

$$C = \lim_{n \rightarrow \infty} \frac{1}{n} \log N(S, n)$$

where  $N(S, n)$  is the number of sequences in  $S$  of length  $n$ .

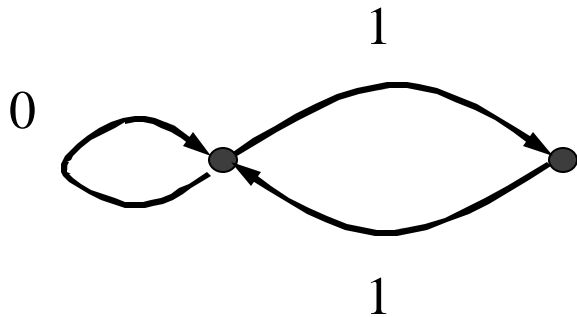
**Theorem [Shannon,1948] :** If there exists a decodable code at rate  $R = m/n$  from binary data to  $S$ , then  $R \leq C$ .

**Theorem [Shannon,1948] :** For any rate  $R = m/n < C$  there exists a block code from binary data to  $S$  with rate  $km:kn$ , for some integer  $k \geq 1$ .

# Computing Capacity: Adjacency Matrices

---

- Let  $A_G$  be the adjacency matrix of the graph  $G$  representing  $S$ .



$$A_G = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

- The entries in  $A_G^n$  correspond to paths in  $G$  of length  $n$ .

# Computing Capacity (cont.)

---

- Shannon showed that, for suitable representing graphs  $G$ ,

$$C = \log \rho(A_G)$$

where  $\rho(A_G) = \max\{|\lambda|: \lambda \text{ is an eigenvalue of } A_G\}$ , i.e., the spectral radius of the matrix  $A_G$ .

- Assigning “transition probabilities” to the edges of  $G$ , the constrained system  $S$  becomes a Markov source  $x$ , with entropy  $H(x)$ . Shannon proved that

$$C = \max H(x)$$

and expressed the maximizing probabilities in terms of the spectral radius and corresponding eigenvector of  $A_G$ .

# *Constrained Coding Theorems*

---

- **Stronger coding theorems were motivated by the problem of constrained code design for magnetic recording.**

**Theorem[Adler-Coppersmith-Hassner, 1983]**

**Let  $S$  be a finite-type constrained system. If  $m/n \notin C$ , then there exists a rate  $m:n$  sliding-block decodable, finite-state encoder.**

**(Proof is constructive: state-splitting algorithm.)**

**Theorem[Karabed-Marcus, 1988]**

**Ditto if  $S$  is almost-finite-type.**

**(Proof not so constructive...)**

# *Distance-Enhancing Codes for Partial Response Channels*

---

- **Beginning in 1990, disk drives have used a technique called partial-response equalization with maximum-likelihood detection, or PRML. In the late 1990's, extensions of PRML, denoted EPRML and EEPRML were introduced.**
- **The performance of such PRML systems can be improved by using codes with “distance-enhancing” constraints.**
- **These constraints are described by a finite set  $D$  of “forbidden differences,” corresponding to differences of channel input sequences whose corresponding outputs are most likely to produce detection errors.**

# Codes that Avoid Specified Differences

---

- The difference between length- $n$  binary words  $u$  and  $v$  is

$$u - v = (u_1 - v_1, \dots, u_n - v_n) \in \{-1, 0, 1\}^n$$

- A length- $n$  code avoids  $D$  if no difference of codewords contains any string in  $D$ .

- Example:  $D = \{++ , +- \}$

Length-2 code:  $C_2 = \{u, v\} = \{00, 10\}$

$$u - v = (-0)$$

# *Capacity of Difference Set $D$*

## [Moision-Orlitsky-Siegel]

---

- **How high can the rate be for a code avoiding  $D$ ?**
- **Define the capacity of the difference set  $D$ :**

$$cap(D) = \log \left[ \lim_{n \rightarrow \infty} (d_n(D))^{1/n} \right],$$

where  $d_n(D)$  is the maximum number of codewords in a (block) code of length  $n$  that avoids  $D$ .

- **Problem: Determine  $cap(D)$  and find codes that achieve it.**

# *Computing $\text{cap}(D)$ : Adjacency Matrices*

---

- Associate to  $D$  a set of graphs and corresponding set  $\Sigma(D)$  of adjacency matrices reflecting disallowed pairs of code patterns:

$$\Sigma = \Sigma(D) = \{A_i : i = 1, \dots, k\}$$

- Consider the set of  $n$ -fold products of matrices in  $\Sigma$  :

$$\Sigma^n = \left\{ \prod_{j=1}^n B_j : B_j \in \Sigma \right\}$$

- Each product corresponds (roughly) to a code avoiding  $D$ .

## *Generalized Spectral Radius $r(S)$*

---

- **Define**

$$r_n(\Sigma) = \sup \left\{ r(A) : A \in \Sigma^n \right\},$$

**the largest spectral radius of a matrix in  $S^n$ .**

- **The generalized spectral radius of  $S$  is defined as:**

$$r(\Sigma) = \limsup_{n \rightarrow \infty} [r_n(\Sigma)]^{1/n}$$

**[Daubechies-Lagarias,1992], cf. [Rota-Strang, 1960]**

# *Computing $\text{cap}(D)$*

## *(cont.)*

---

**Theorem[Moision-Orlitsky-Siegel, 2001]**

**For any finite difference set  $D$ ,**

$$\text{cap}(D) = \log r(\Sigma(D)) .$$

**Recall formula for the capacity of a constrained system  $S$**

$$C = \log ?(A_G) .$$

- **Computing  $\text{cap}(D)$  can be difficult, but a constructive bounding algorithm has yielded good results.**

## *A Real Example: EEPRML*

---

- Codes can improve EEPRML performance by avoiding

$$D = \{0 + - + 0\}$$

- Codes satisfying the following constraints avoid  $D$ :

»  $F = \{101, 010\}$                        $C = 0.6942\dots$

»  $F = \{101\}$                                $C = 0.8113\dots$

»  $F = \{0101, 1010\}$                        $C = 0.8791\dots$       MTR [Moon, 1996]

- What is  $cap(D)$ , and are there other simple constraints with higher capacity that avoid  $D$  ?

# *EEPRML Example*

## *(cont.)*

---

- For  $D = \{0+-+0\}$  ,

$$0.9162 \text{ } \mathfrak{L} \text{ } \text{cap}(D) < 0.9164.$$

- The lower bound, conjectured to be exactly  $\text{cap}(D)$ , is achieved by the “time-varying MTR (TMTR)” constraint, with finite periodic forbidden list:

$$F = \left\{ 1010^{\text{odd}}, 0101^{\text{odd}} \right\}.$$

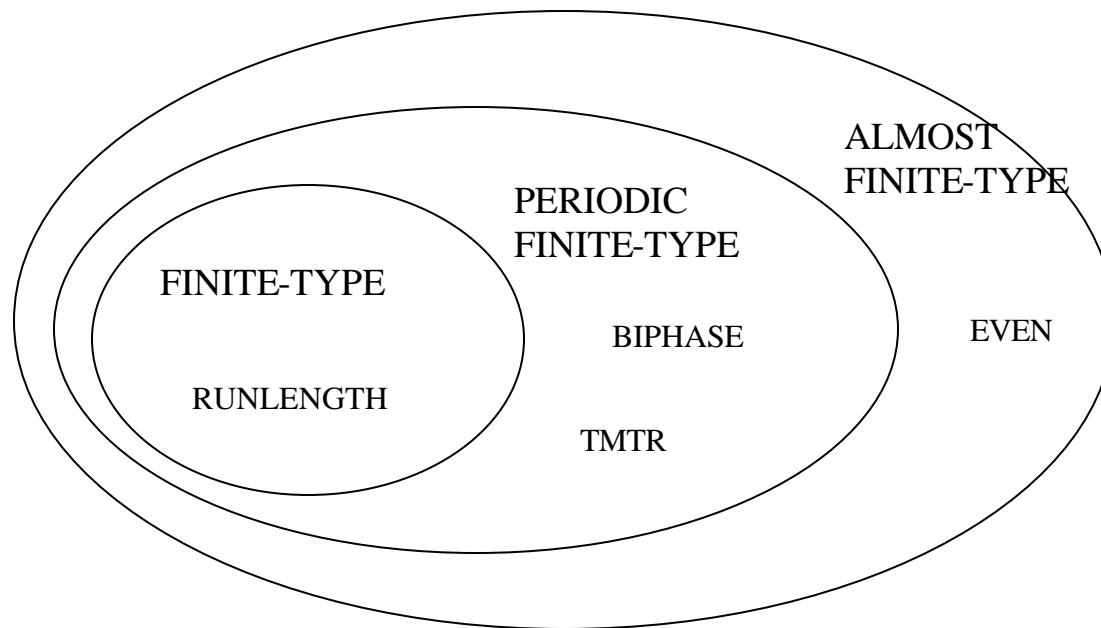
- Rate 8/9, TMTR code has been used in commercial disk drives [Bliss, Wood, Karabed-Siegel-Soljanin].

# *Periodic Finite-Type Constraints*

---

- The TMTR (and biphase) constraint represent a new class of constraints, called “periodic finite-type,” characterized by a finite set of periodically forbidden words.

[Moision-Siegel, ISIT 2001]



## *Other Storage-Related Research*

---

- **Page-oriented storage technologies, such as holographic memories, require codes generating arrays of bits with 2-dimensional constraints. This is a very active area of research.**

**[Wolf, Shannon Lecture, ISIT 2001]**

- **There has been recent progress related to computing the capacity of noisy magnetic recording (ISI) channels,**

$$\mathbf{C} = \mathbf{Max} (\mathbf{H}(\mathbf{x}) - \mathbf{H}_y(\mathbf{x})) .$$

**[Arnold-Loeliger, Arnold-Vontobol, Pfister-Soriaga-Siegel, Kavcic, 2001]**

## *Conclusion*

---

- **The work of Claude Shannon has been a key element in the “miraculous” progress of modern information storage technologies.**
- **In return, the ongoing demand for data storage devices with larger density and higher data transfer rates has “miraculously” continued to inspire new concepts and results in information theory.**